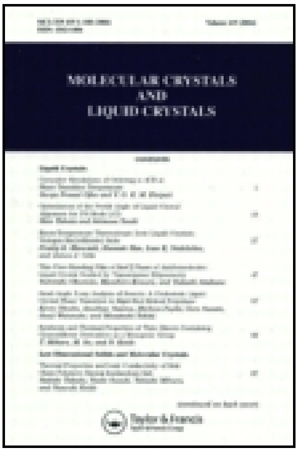

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Theoretical Study of Convective Motions in Liquids and Liquid Crystals Induced by Laser Radiation with Gaussian Cross Distribution of Intensity

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Theoretical Study of Convective Motions in Liquids and Liquid Crystals Induced by Laser Radiation with Gaussian Cross Distribution of Intensity

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Convective motion generated by Gaussian laser beam impinging normally on the sample was theoretically discussed in this study for both isotropic liquids and liquid crystals. Heat transfer, Navier-Stokes and director equations were solved numerically and corresponding distributions of heat, velocity and director were obtained. Rayleigh-Benard and Benard-Marangoni convections in liquid crystals were also discussed. It was shown that theoretical calculations are in a strong agreement with experiments done previously.

Keywords Convection; Gaussian beam; liquid crystal; director

Introduction

Laser generated instabilities in isotropic and anisotropic liquids have recently became an interesting subject for both theoretical and experimental study. The first systematic studies of convection in liquids were published in [1–3] studies. Many studies are dedicated to convection in liquid layer heating from below [4–9]. Theoretical and experimental studies of gravitational and thermocapillar mechanisms of convection in both isotropic and anisotropic mediums due to absorption of radiation with spatial-periodic distribution of intensity were carried out in [10]. Convective motions reorienting director in liquid crystal can be visualized by means of several optical methods allowing us to create new practical applications. Theoretical study of forced convection and consequent orientation of molecules in nematic liquid crystal with one open boundary was given in [11]. Experimental study of convection in nematic liquid crystal generated by a Gaussian laser beam is presented in [12].

The aim of current research is to study convection motion in a horizontal layer of incompressible liquids and nematic liquid crystal (NLC) induced by Gaussian laser beam impinging normally to the wall of the cell. Both Rayleigh-Benard and Benard-Marangoni convections were studied theoretically by solving flow, heat transfer and director equations simultaneously using numerical methods.

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The present article is organized as follows. In Sec. 1, solution of Navier-Stokes and heat transfer equations in the case of Gaussian laser beam for isotropic liquid is presented. Sec. 2 and Sec. 3 are dedicated to the obtainment of director distribution in a nematic liquid crystal homeotropic cell due to Rayleigh-Benard and Benard-Marangoni convections. Sec. 4 is the conclusion.

1. Laser Induced Convection in Isotropic Liquids

Here we consider a Gaussian beam

$$\mathrm{l(r)} = \frac{2\mathrm{P}}{\pi \mathrm{a}^2} \exp\left(- \frac{2\mathrm{r}^2}{\mathrm{a}^2} \right), \tag{1}$$

which impinges normally on the bottom wall of the cell (P is the power of the laser beam, a – beam size, which is 0.1 mm in all calculations). Laser beam passing through the cell heats it due to the sample absorption. This heating process in conjunction with the gravitation field or surface tension temperature dependence induces instabilities, thus, convective motions in the cell. Heat generation and transport in the cell is described by the heat transfer equation

$$\rho \mathrm{c} \frac{\partial \mathrm{T}}{\partial \mathrm{t}} + \rho \mathrm{c v} \cdot \nabla \mathrm{T} = \nabla \cdot (\mathrm{k} \nabla \mathrm{T}) + \mathrm{Q(r)} \tag{2}$$

$$\mathrm{Q(r)} = \alpha \mathrm{l(r)}, \tag{3}$$

where c is the specific heat, ρ – density, k – heat conductivity and Q(r) – heat generated per unit time in unit volume of the cell.

Convective motion in a medium is described by Navier-Stokes and continuity equations

$$\rho \left(\frac{\partial \vec{\mathrm{v}}}{\partial \mathrm{t}} + (\vec{\mathrm{v}} \vec{\nabla}) \vec{\mathrm{v}} \right) = \vec{\nabla} p + \eta \Delta \vec{\mathrm{v}} + \rho \vec{g} \tag{4}$$

$$\mathrm{div} \vec{\mathrm{v}} = 0, \tag{5}$$

where v is the velocity, p – pressure, η – dynamic viscosity, ρ – density and g – standard gravity. It is worth to mention that $\vec{\mathrm{v}}(\mathrm{t}=0) = \vec{\mathrm{v}}_0 = 0$, $\mathrm{T}(\mathrm{t}=0) = \mathrm{T}_0 = \mathrm{const}$, $\mathrm{p}(\mathrm{t}=0) = \mathrm{p}_0 = \mathrm{p}(\mathrm{z}=0) - \rho_0 \mathrm{gz}$ before the laser beam is switched on. Temperature, pressure, and consequently the density of the medium suffer changes after switching on the laser

$$\mathrm{T}' = \mathrm{T} - \mathrm{T}_0, \mathrm{p}' = \mathrm{p} - \mathrm{p}_0, \rho = \rho_0 + \rho' = \rho_0(1 - \beta \mathrm{T}'), \tag{6}$$

where $\beta = -\frac{1}{\rho}(\frac{\partial \rho}{\partial \mathrm{T}})$ is the volume expansion coefficient. Thus, Navier-Stokes equation is

$$\frac{\partial \vec{\mathrm{v}}}{\partial t} + (\vec{\mathrm{v}} \vec{\nabla}) \vec{\mathrm{v}} = -\frac{\vec{\nabla} p}{\rho} + \upsilon \Delta \vec{\mathrm{v}} - \vec{g} \beta T', \tag{7}$$

where $\upsilon = \frac{\eta}{\rho}$ is the kinematic viscosity. As our problem has a central symmetry, it is convenient to solve these equations in cylindrical coordinates system taking into account that all variables are independent of $\varphi(\frac{\partial}{\partial \varphi} = 0)$. Based on the experimental results it is assumed that $\mathrm{v}_\varphi = 0$, because there is no flow in this direction [12]. Navier-Stokes equations

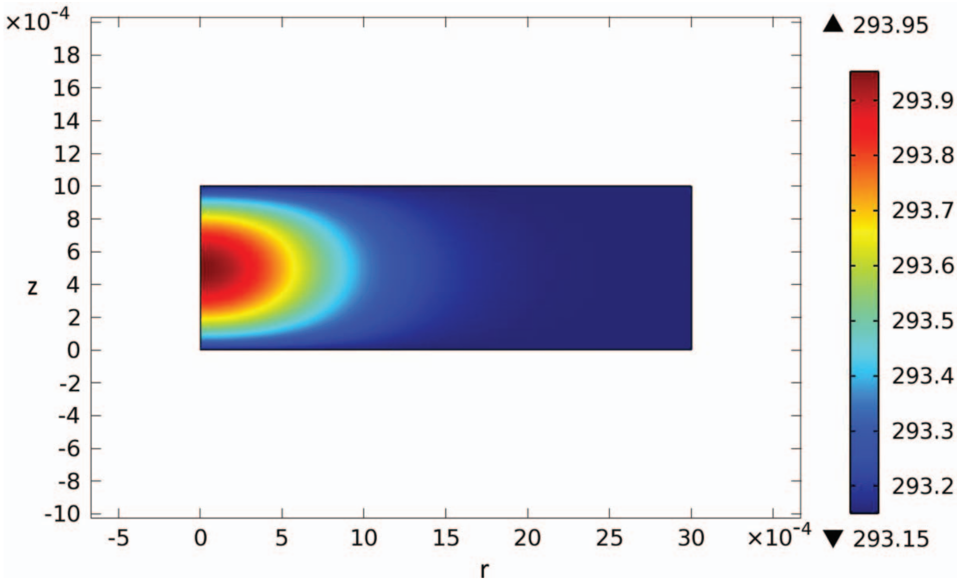

Figure 1. Heat generated in a liquid with the following characteristics—absorption coefficient is $3\ cm^{-1}$, laser power $P = 40$ mW, thermal conductivity of medium $k = 2.35$ W/m·K, waist size is 1 mm and density—1380 kg/m^3.

for z and r components of velocity are

$$\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + v_z\frac{\partial v_r}{\partial z} = -\frac{1\partial p}{\rho\partial r} + \frac{\upsilon}{\rho}\Delta v_r \tag{8}$$

$$\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + v_z\frac{\partial v_z}{\partial z} = -\frac{1\partial p}{\rho\partial z} + \frac{\upsilon}{\rho}\Delta v_z - g\beta T'. \tag{9}$$

The sample is placed in a thermostat, i.e. temperature on both boundaries of the medium is fixed at ambient temperature $T(r, z = 0) = T(r, z = L) = 293.5$ K. The solution of heat transfer and flow equations for $L = 1$ mm thick medium are depicted in Figs. 1 and 2. COMSOL Multiphysics softwere has been used to get the numerical solution. It can be seen in these plots that temperature distribution is similar to Gaussian beam intensity distribution. Velocity is high in those points, where temperature is high, because the last term in Navier-Stokes equation which is the cause of convective motion, depends on temperature linearly. Maximum velocity value is about 1.62 μm/sec.

2. Laser Induced Convection in Nematic Liquid Crystals

2.1. General Formulation of the Problem

Convective motions reorient the director vector whose dynamics can be obtained by solving Euler-Lagrange-Rayleigh system of equations

$$\prod_{ij}\left[\frac{\delta R}{\delta(\partial n_j/\partial t)} - \frac{\partial}{\partial x_k}\frac{\delta R}{\delta(\partial^2 n_j/\partial x_k\partial t)} + \frac{\delta F}{\partial n_j} - \frac{\partial}{\partial x_k}\frac{\delta F}{\delta(\partial n_j/\partial x_k)}\right] = 0. \tag{10}$$

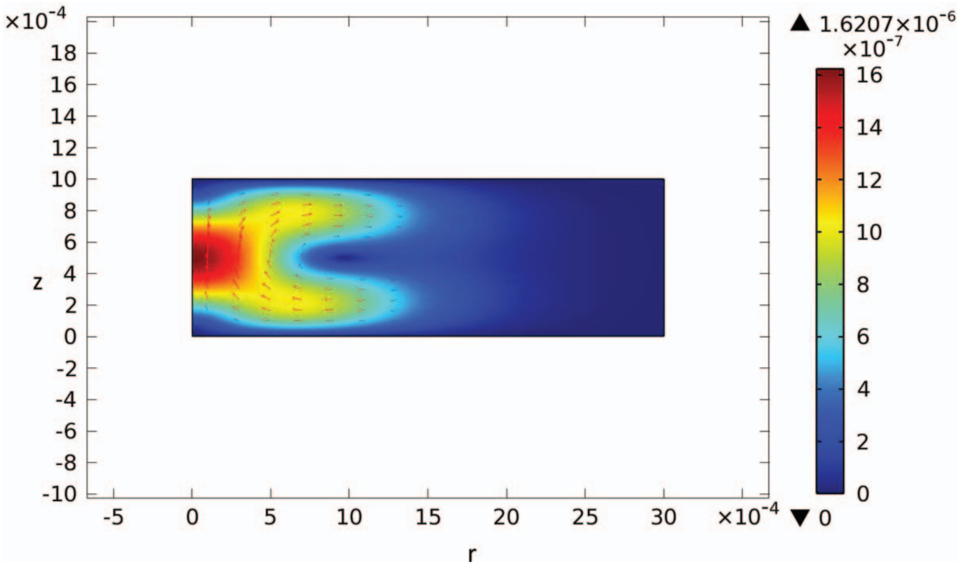

Figure 2. Temperature distribution and velocity (m/s) distribution for L = 1 mm thick cell. Red arrows show the direction of velocity vector in every point of the medium.

where $\prod_{ij} = \partial_{ij} - \eta_i \eta_j$ is an operator ensuring the fulfillment of the condition of normalization $n^2 = 1$, R – dissipation function and F – density of the free energy, which have the following form

$$R = \mu_1 d_{ij} d_{ij} + \mu_2 \eta_i N_j d_{ij} + \mu_3 N_i N_i + \mu_4 \eta_i \eta_j d_{ik} d_{kj} + \mu_5 \eta_i \eta_j \eta_k \eta_m d_{ij} d_{km}$$

$$d_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right), \vec{N} = \frac{d\vec{n}}{dt} + \frac{1\left[\vec{n}\,\mathrm{rot}\,\vec{v}\right]}{2} \tag{11}$$

$$F = \frac{1}{2} K \frac{\partial n_i}{\partial x_j}\frac{\partial n_i}{\partial x_j}, \tag{12}$$

where μ_i are constants connected with α_i coefficients of Leslie. One-constant approximation for the density of free energy is used here ($K_1 = K_2 = K_3 = K$) to simplify our calculations. 5CB nematic liquid crystal homeotropic cell was used as a sample (Fig. 3).

We assume that director deviation is small and all second order members in the formulas can be neglected. Taking into account the homeotropic boundary conditions for the director at z = 0 and z = L, one finds $\delta n_z = 0$. Consequently, the perturbed director field can be sought as $\delta\vec{n} = (\delta n_r, 0, 0)$ in the (r, φ, z) cylindrical coordinates system. Combining equations (8,9,10), the equation describing $\delta n_r(r, z)$ is given by

$$K\nabla\delta n_r = \frac{1}{2}\mu_2\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right) + 2\mu_3\frac{d\delta n_r}{dt} - \mu_3\left(\frac{\partial v_\rho}{\partial z} + \frac{\partial v_z}{\partial r}\right) \tag{13}$$

This equation should be solved in conjunction with boundary conditions typical for a specific problem. Hereafter, two cases have been explored: the Rayleigh – Benard (RB) and the Benard – Marangoni (BM).

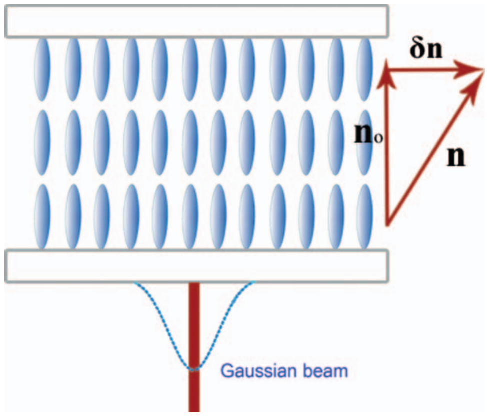

Figure 3. Gaussian laser beam impinges normally on the bottom wall of 5CB nematic liquid homeotropic cell.

2.2. RB Convection

Let us discuss the convection due to density gradient in the gravitation field in the cell with two closed boundaries. It is assumed that the influence of anisotropy on velocity and temperature distributions is negligible. Solving the above-mentioned equation with given boundary conditions $\delta n_r(r, z = 0) = \delta n_r(r, z = L) = 0$, we will get director distribution. Steady-state solution for director distribution is shown in Fig. 4.

2.3. BM Convection

In this case convection emerges due to a temperature dependence of surface tension and the equation describing equilibrium of forces on the free surfaces should also be taken into

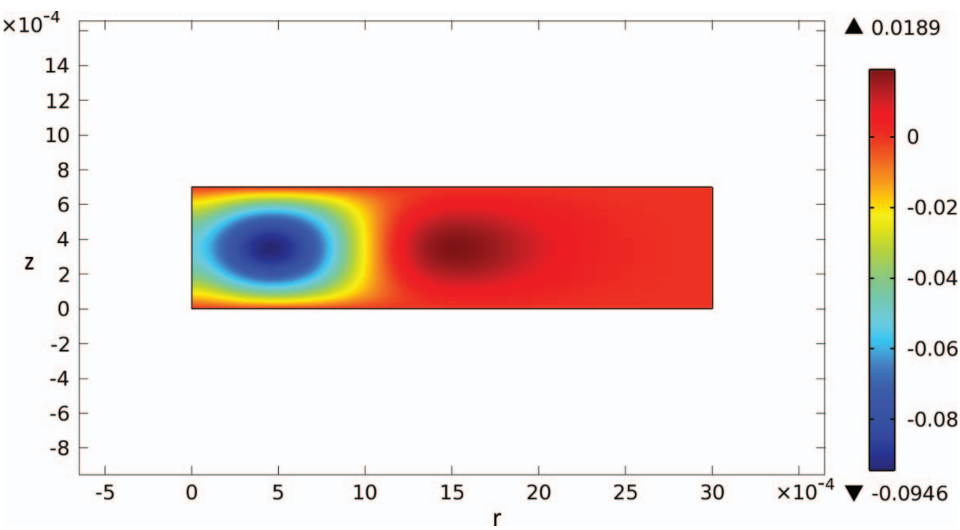

Figure 4. Distribution of director in homeotropic cell for P = 40 mW Gaussian laser beam.

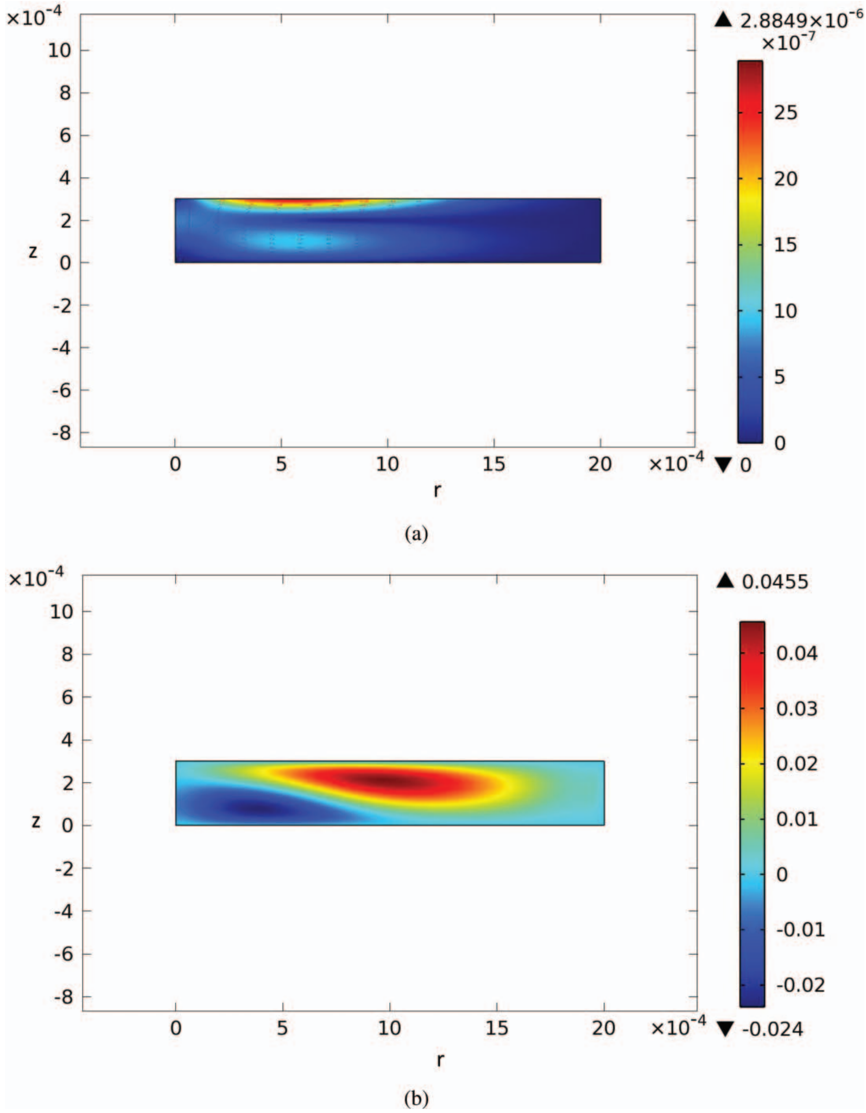

Figure 5. a) Velocity distribution and b) director distribution in the 5CB nematic liquid crystal cell for 40 mW laser beam.

account. Viscosity stress tensor on the free surface is

$$\sigma_{zr} = \eta \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right) \tag{14}$$

It's assumed that surface tension depends on temperature linearly. This dependence has the following form

$$\sigma = \sigma_0 - \sigma' T,$$

where σ' is temperature derivative of the surface tension.

It's easy to get from these two last equations that

$$\eta \frac{\partial v_r}{\partial z} + \sigma' \frac{\partial T}{\partial r} \tag{15}$$

Benard-Marangoni convection is typical for thin layers where density gradient can't be the main cause of convection. It is worth to mention that in all cases two mechanisms of convection are present, if we consider a medium with an open surface. Ratio of Rayleigh number to Marangoni number describing the relative importance of two effects [14] involved in BM convection is

$$\frac{R}{M} = \left(\frac{\rho\beta g}{\sigma'}\right) L^z, \tag{16}$$

where ρ is the density, β – coefficient of thermal volume expansion, g – acceleration due to gravity and L – thickness of the layer. Boussinesq approximation is used to solve this flow problem, i.e. characteristics of the medium are assumed to vary very slowly. So, the expression in brackets is constant and the prevalence of one of these mechanisms depends on the thickness of the layer. Thus, Marangoni convection can be observed in thin layers (usually smaller than 1mm). In our case the value of the expression in brackets is about 10^{-2}, thus, the convection is due to temperature dependence of surface tension.

Solutions of Navier-Stokes and director equations when taking into account this contribution on the free surface are presented in Fig. 5a and Fig. 5b.

Maximum velocity in the case of Marangoni convection is $\approx 3\ \mu$m/s.

Conclusion

Convective motion in liquids and liquid crystals generated by Gaussian laser beam has been discussed in the present study for the first time. It was shown that velocity magnitude for both Rayleigh-Benard and Benard-Marangoni convections is about several micrometers per second. Laser power in calculations has been deliberately taken small in order not to get high temperatures in the medium avoiding nematic-isotropic transition. In our calculations temperature rise is about 1K. As it can be seen in plots presented above, convection appears in the direction of red arrows. All these results are in a good agreement with previously done experiments [12].

Acknowledgment

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